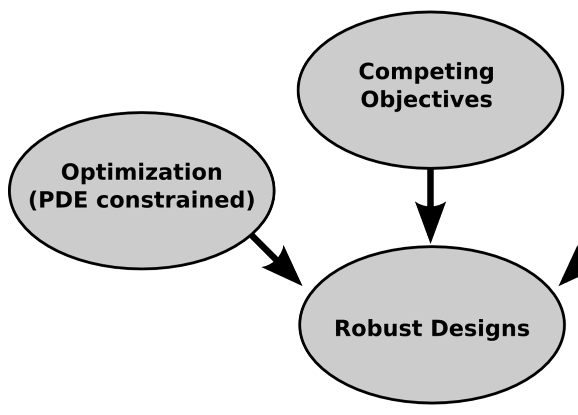
Robust Multi-Objective Airfoil Design: Results and Challenges

Lisa Kusch, AG Scientific Computing, TU Kaiserslautern

Overview: Robust Multi-Objective Optimization



The general research idea is to find an efficient and successful method for robust multi-objective optimization involving different topics, that have to be combined.

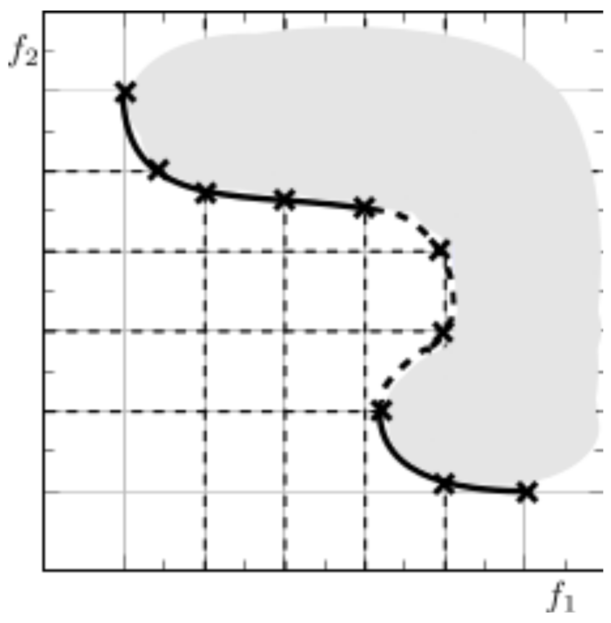
Multi-Objective Optimization (MOO)

The aim of multi-objective optimization algorithms is to find a representative subset of **Pareto optimal** solutions. Multi-objective robust design is mainly treated in an evolutionary context [1]. We make use of the **equality-constraint method** and the ε-**constraint method** [2] enabling the use of deterministic methods for single-objective optimization.

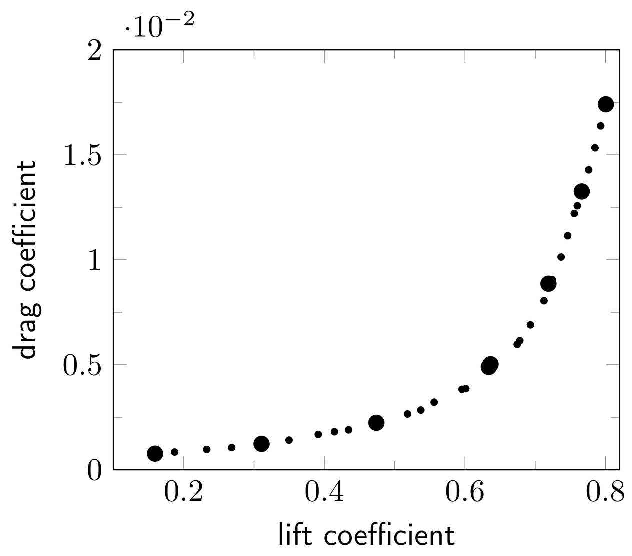
The concept is to optimize one objective function while imposing equality or inequality constraints on the remaining objective functions. The constraints as well as the objective function to be optimized. The resulting problem for k objective functions is

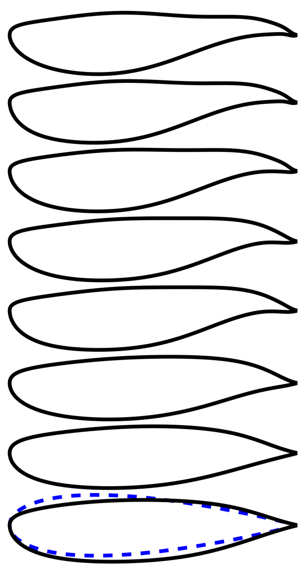
$$\min_{\boldsymbol{y},\boldsymbol{u}} \quad f_s(\boldsymbol{y},\boldsymbol{u}) \qquad \text{s.t.} \quad \boldsymbol{c}(\boldsymbol{y},\boldsymbol{u}) = \boldsymbol{0},\ f_i(\boldsymbol{y},\boldsymbol{u}) \leq f_i^{(j)} \quad \forall\, i \in \{1,...,k\} \quad i \neq s. \tag{1}$$

All unique solutions to (1) are globally Pareto optimal for any upper bound.



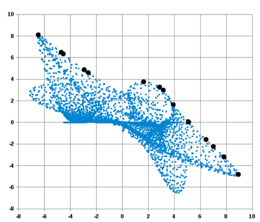
Scanning procedure with equality-constraint method





The Pareto-optimal front for the application is shown on the left. The underlying multi-objective optimization problem reads

$$\min_{\boldsymbol{y},\boldsymbol{u}} \ (c_d(\boldsymbol{y},\boldsymbol{u}), -c_l(\boldsymbol{y},\boldsymbol{u})) \quad \text{s.t.} \ \boldsymbol{c}(\boldsymbol{y},\boldsymbol{u}) = \boldsymbol{0},\ c_m(\boldsymbol{y},\boldsymbol{u}) = 0.0,\ d(\boldsymbol{y},\boldsymbol{u}) = 0.12.$$



Test function (hybrid: GA, Ipopt)

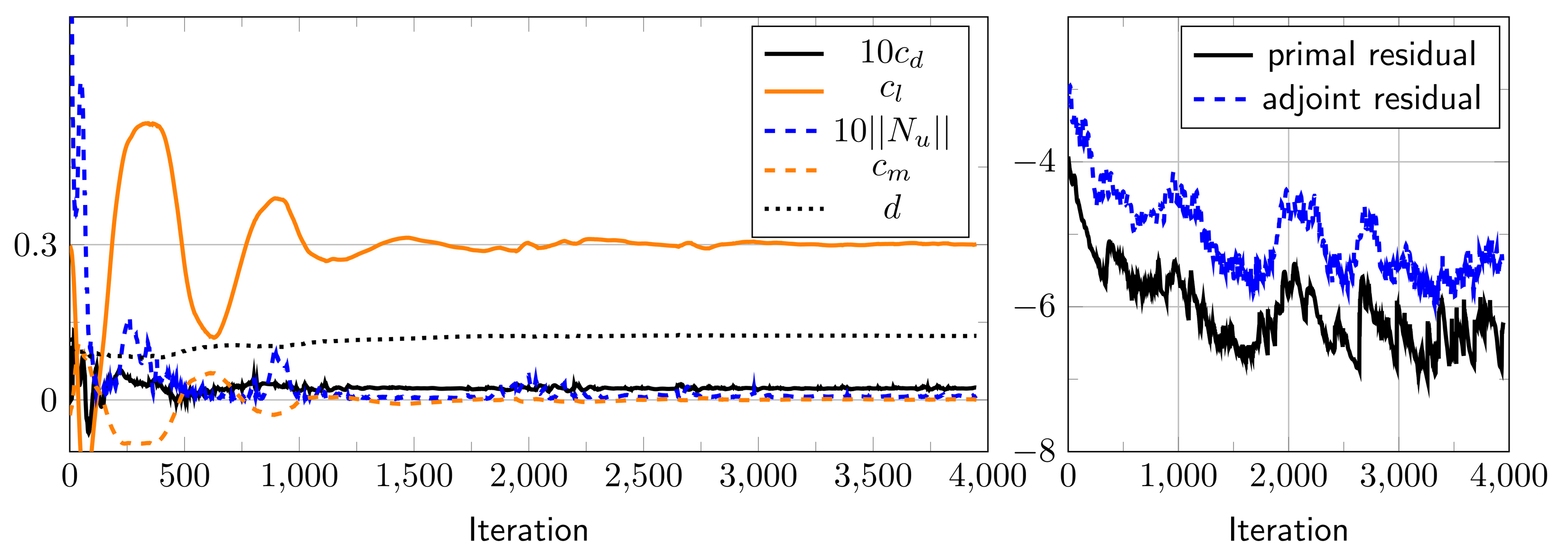
Challenge for specific applications: To enhance the chance of finding a global optimum the resulting single-objective optimization problems can be solved using a **hybrid approach** (e.g by combining a genetic algorithm with a gradient-based method).

PDE Constrained Optimization

We make use of the **one-shot method** with additional equality constraints [3] using algorithmic differentiation. Instead of doing a nested optimization, the idea is to **simultaneously** obtain **primal and adjoint feasibility**, as well as **optimality**. Assuming that the state equation can be transformed into a contractive fixed point form $G(y,u) = y$, we iterate

$$\begin{aligned}
\boldsymbol{y}_{k+1} &= \boldsymbol{G}(\boldsymbol{y}_k, \boldsymbol{u}_k) && \text{(primal iteration)}\\
\boldsymbol{u}_{k+1} &= \boldsymbol{u}_k - \mathrm{B}_k^{-1} \mathrm{N}_u(\mathrm{y}_k, \mathrm{u}_k, \hat{\boldsymbol{\lambda}}_k)^\top && \text{(design iteration)}\\
\boldsymbol{\lambda}_{k+1} &= N_{\boldsymbol{y}}(\boldsymbol{y}_k, \boldsymbol{u}_k, \hat{\boldsymbol{\lambda}}_k)^\top && \text{(adjoint iteration)}\\
\boldsymbol{\mu}_{k+1} &= \boldsymbol{\mu}_k - \check{\mathrm{B}}_k^{-1} \mathrm{h}(\mathrm{y}_k, \mathrm{u}_k) && \text{(augmented iteration),}
\end{aligned} \tag{2}$$

where $N(\boldsymbol{y},\boldsymbol{u},\hat{\boldsymbol{\lambda}}) = f(\boldsymbol{y},\boldsymbol{u}) + \boldsymbol{G}(\boldsymbol{y},\boldsymbol{u})^\top \boldsymbol{\lambda} + \boldsymbol{h}(\boldsymbol{y},\boldsymbol{u})^\top \boldsymbol{\mu}$.



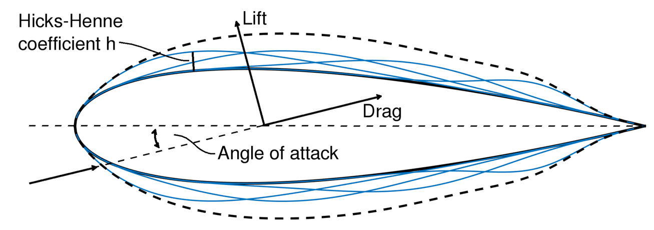
Optimization history: One-shot with equality constraints using algorithmic differentiation (AD) and a constant preconditioner $\check{B}$

Challenges: Inequality constraints and the choice of the constraint multiplier preconditioner.

Aerodynamic Shape Optimization in SU2

The concepts are applied to aerodynamic shape optimization using the open-source multi-physics package SU2 and

- drag coefficient c_d and lift coefficient c_l as competing objectives,
- 38 Hicks-Henne design variables
- constraints on moment c_m and thickness d,
- a steady, transonic Euler flow and the
- AD-based discrete adjoint solver in SU2 [4].



Challenge: Application to multi-disciplinary optimization in the context of fluid-structure interaction (shape optimization, if possible topology optimization).

Robustness in MO Context

We consider a solution robust if it is not very sensitive to aleatory uncertainties $\boldsymbol{\omega}$. In multi-objective design **different concepts** for finding robust solutions can be considered:

- expectation-based approach:

$$\min_{\boldsymbol{y},\boldsymbol{u}} \ \mathsf{Exp}(\mathbf{F}(\boldsymbol{y},\boldsymbol{u},\boldsymbol{x}(\boldsymbol{\omega}))) \tag{I}$$

or

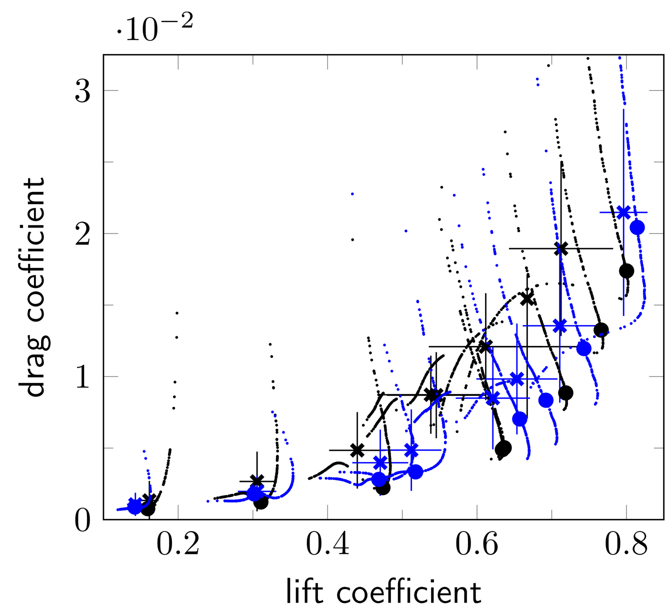
$$\min_{\boldsymbol{y},\boldsymbol{u}} \mathbf{F}(\boldsymbol{y},\boldsymbol{u},\bar{x}) \quad \text{s.t.} ||\mathsf{Exp}(\mathbf{F}) - \mathbf{F}|| \leq \nu, \tag{II}$$

- expectation-variance-based approach:

$$\min_{\boldsymbol{y},\boldsymbol{u}} \ (\mathsf{Exp}(\mathbf{F}), \mathsf{Var}(\mathbf{F})) \tag{III}$$

or

$$\min_{\boldsymbol{y},\boldsymbol{u}} \mathbf{F}(\boldsymbol{y},\boldsymbol{u},\bar{x}) \quad \text{s.t.} ||\mathsf{Var}(\mathbf{F})|| \leq \mu, \tag{IV}$$



The difference to single-objective robust optimization is that we have

- a combined effect of uncertainties in objective space, and
- we have to consider a set of robust solutions.

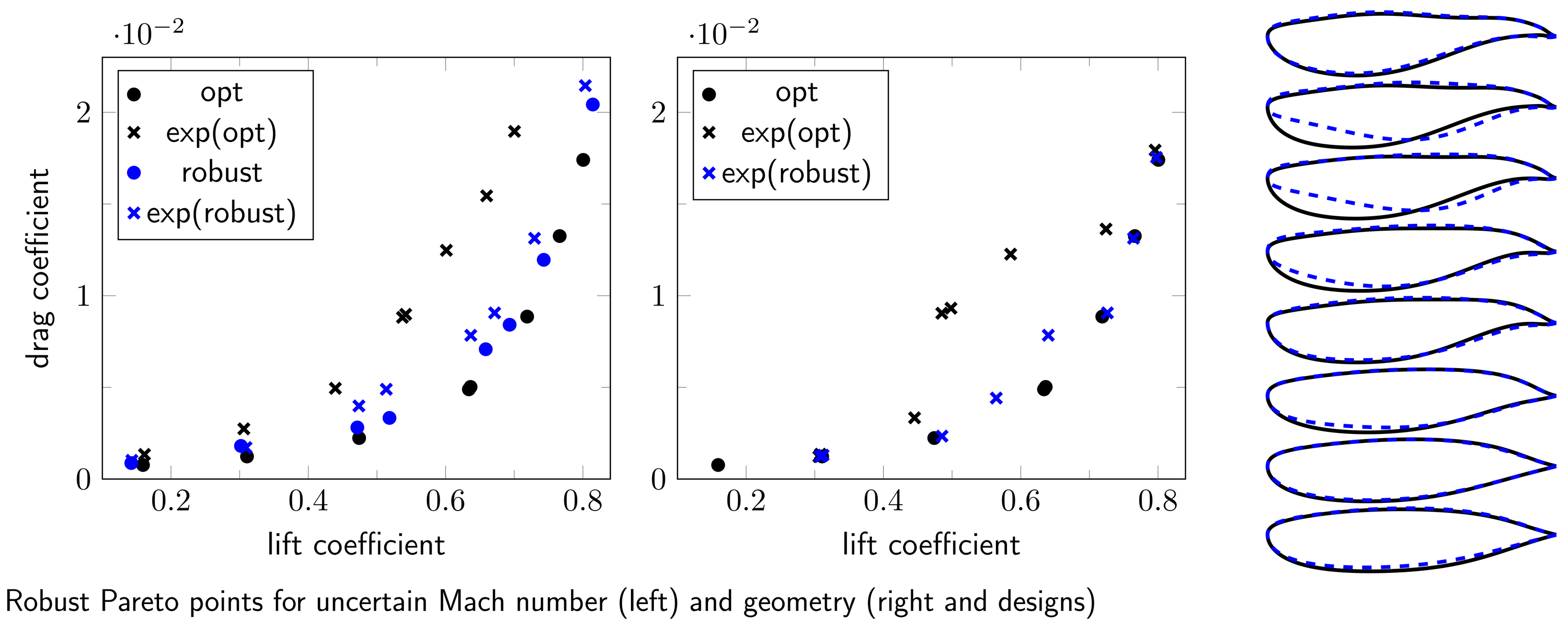
For aerodynamic shape optimization we consider robustness of type (I). The figure shows the effect of an uncertain Mach number $Ma \sim N(0.8, 0.01)$ in objective space.

Challenge: Is it sufficient to look at statistical quantities? What about gains and losses?

Uncertainty Quantification

We make use of a **non-intrusive polynomial chaos** approach, in which the stochastic objective function is expanded in terms of polynomials Φ_i that are orthogonal with respect to the density function of the input random variables $\boldsymbol{x}(\boldsymbol{\omega})$:

$$f(\boldsymbol{y},\boldsymbol{u},\boldsymbol{x}(\boldsymbol{\omega})) \approx \sum_{i=1}^{M} \hat{f}_i(\boldsymbol{y},\boldsymbol{u}) \Phi_i(\boldsymbol{x}),\ \ \hat{f}_i(\boldsymbol{y},\boldsymbol{u}) = \gamma_i \mathbb{E}(f(\boldsymbol{y},\boldsymbol{u},\boldsymbol{x})\Phi_i) \approx \sum_{k=1}^{Q} w_i f(\boldsymbol{y},\boldsymbol{u},\boldsymbol{x}_i).$$



Robust Pareto points for uncertain Mach number (left) and geometry (right and designs)

The uncertain geometry (perturbed in normal direction by random process with expected function and covariance function) is modelled with the help of a truncated Karhunen-Loeve expansion and the computational effort is reduced by using sparse grids for the Gauss-Hermite quadrature points (see [5]).

[1] K. Deb, H. Gupta, Searching for Robust Pareto-Optimal Solutions in Multi-Objective Optimization. Lecture Notes in Computer Science, 3410, 150–164, 2005.

[2] S. A. Marglin, Public investment criteria: Benefit-cost analysis for planned economic growth. M.I.T Press, Cambridge, Massachusetts, 1967.

[3] A. Walther, N. Gauger, L. Kusch, and N. Richert, On an extension of one-shot methods to incorporate additional constraints, Optim. Methods Softw. 31 (2016), 494–510.

[4] T. Albring, M. Sagebaum, and N. Gauger, Development of a consistent discrete adjoint solver in an evolving aerodynamic design framework, AIAA 2015-3240 (2015).

[5] C. Schillings, S. Schmidt, V. Schulz, Efficient shape optimization for certain and uncertain aerodynamic design. Computers and Fluids, 76, 78—87, 2011.