Robustness Measures for Multi-Objective Robust Design

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Summary

A significant step to engineering design is to take into account uncertainties and to develop optimal designs that are robust with respect to perturbations. Furthermore, when multiple optimization objectives are involved it is important to define suitable descriptions for robustness. We introduce robustness measures for robust design with multiple objectives that are suitable for considering the effect of uncertainties in objective space. A direct formulation and a two-phase formulation based on expected losses in objective space are presented for finding robust optimal solutions. We apply the two-phase formulation to the robust design of an airfoil. Fluid mechanical quantities are optimized under the consideration of aleatory uncertainties. The uncertainties are propagated with the help of the non-intrusive polynomial chaos approach. The resulting multi-objective optimization problem is solved with a constraint-based approach, that combines adjoint-based optimization methods and evolutionary methods evaluated on surrogate models.

Keywords: *robust design, multi-objective optimization, adjoint method, hybrid method; robustness measure*

1 Introduction

Multi-objective optimization and robust design are two well-established fields of research. Especially, in engineering applications it is important to optimize for different conflicting criteria for example cost and quality aspects. Here, the aim is to find a set of solutions that fulfill the concept of Pareto optimality. A feasible design x is Pareto optimal if it is non-dominated, i.e. there does not exist any feasible design $\bar{x}$ such that $f_i(\bar{x}) \leq f_i(x)$ for every objective function f_i with $i \in \{1,...,k\}$ and $f_j(\bar{x}) < f_j(x)$ for at least one $j \in \{1,...,k\}$. The image of the Pareto optimal set in objective space is denoted as the Pareto optimal front. We distinguish between scalarization approaches and direct approaches for multi-objective optimization. In scalarization approaches, for example constraint-based methods, the problem is transformed into several single-objective optimization problems, that can be solved efficiently using hybrid methods combining gradient-based optimization methods and global search methods.

Another significant step towards realistic multi-objective design is to take into account uncertainties for finding robust optimal solutions. Robust optimal solutions are solutions, that are optimal and robust with respect to perturbations. Most of the robustness measures for multi-objective optimization are inspired by single-objective robustness definitions based on statistical quantities. We distinguish between expectation-based and variance-based measures. Also, the quantities can either be objectives or set as additional constraints. Two expectation-based measures were for example proposed by Deb and Gupta[1] and adapted for aerodynamic shape optimization.[2] Furthermore, there exist methods specifically tailored for multi-objective optimization problems. The application to evolutionary multi-objective optimization enables the use of a probability of dominance or an expected fitness function.[3] For a local sensitivity analysis a local sensitivity region[4] can be used in objective space.

In Section 2 expected losses are introduced as a measure for robustness when considering multiple objectives and two different approaches to robust optimal design are presented that both result in a multi-objective optimization problem. The constraint method for solving the multi-objective optimization problems is presented in Section 3. The proposed strategy is applied for finding robust optimal solutions in aerodynamic shape optimization with aleatory uncertainties in Section 4.

2 Robust Design

2.1 Robustness Measures

In the following we introduce a measure for robustness, that can be used in a scalarization approach, and account for effects in objective space. The general idea is to measure the expected distance of an outcome from the deterministic Pareto optimal front. Using the Pareto optimal front we can

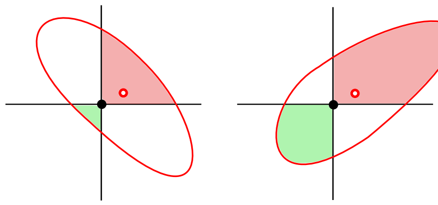

Figure 1: Two different probability regions in objective space

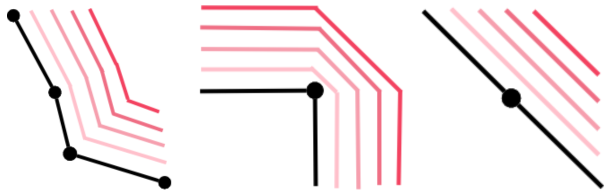

Figure 2: Signed distance function for two-phase approach (left), expected possible losses (middle), linear approximation (right)

state if an outcome of random samples is better or worse (corresponding to gains and losses in objective space).

Figure 1 shows the contours for a fixed probability, that we will refer to as probability region, for two designs. Both designs have a similar deterministic value (black dot). Also, the variances and the expected value (circle) are similar. Nevertheless, one would prefer the left design over the right one since more outcomes are non-dominated by outcomes for the right design. Additionally, the fact that the gains outweigh the losses for the right design shows that a robustness measure should be defined using a loss function based on the distance to the deterministic value or to the Pareto optimal front. Note that the points found in the lower left region are definitely better and the points in the upper right region definitely worse in comparison to the deterministic outcome.

We propose two approaches to describe robustness with the help of losses in objective space. In both approaches the expected losses are constrained by a prescribed upper bound.

2.2 *Two-Phase Approach*

In the two-phase approach we assume that a given set of Pareto optimal points has been determined for the deterministic optimization problem in a first phase. Additionally we assume to have an approximation of the Pareto optimal front in objective space, e.g. by means of splines in the two-dimensional case or by the help of other sophisticated interpolation methods for higher dimension. Note that the approximation can become non-trivial for disconnected Pareto optimal fronts, although a distance to the front can be defined. We denote the representation of the Pareto optimal front as ϕ_0.

The expected losses can be expressed by means of a signed distance function δ, that can be defined by using a level-set method with zero level set ϕ_0. Given a vector of random input variables $z(\omega)$ depending on the uncertainties ω, the corresponding optimization problem to be solved in the second step is

$$\begin{aligned} \min_{y,u} \quad & \mathbf{F}(y,u,\bar{z}) \\ \text{s.t.} \quad & c(y,u,\bar{z}) = 0, \\ & \mathrm{Exp}(\max(0, \delta(\mathbf{F}(y,u,z(\omega)), \boldsymbol{\phi_0})) \leq \delta_{max}. \end{aligned} \tag{1}$$

The variables y and u are the state and design variables that fulfill the state equation $c(y,u) = 0$. The minimization of the objective function vector $\mathbf{F}$ with objective functions F_i for $i = 1,\ldots k$ has to be understood component-wise. The evaluation at $\bar{z}$ denotes the deterministic case where z is not a random variable but the value prescribed when not considering any uncertainties. For reasons of clarity we will omit to include the dependency on y and u in the following definitions.

2.3 *Direct Approach*

In the direct approach the deterministic Pareto optimal front is not given. Instead, the local distance of the samples to the current deterministic value for $\bar{z}$ is used to describe losses. When not considering only losses, this approach is similar to the constrained expectation-based approach.[1] Different assumptions for the local estimation of losses can be made. When considering expected possible losses we may formulate the optimization constraint as

$$\sum_{i=1}^{k} \mathrm{Exp}(\max(0, F_i(z(\omega)) - F_i(\bar{z}))) \leq \mu_1. \tag{2}$$

Another assumption is to approximate the losses based on a local linear approximation of the Pareto optimal in the current deterministic outcome. The local front can then be represented as the zero level set of $\phi = \sum_{i=1}^{k} F_i(z(\omega)) - F_i(\bar{z})$. The corresponding optimization constraint is

$$\mathrm{Exp}(\max(0, \sum_{i=1}^{k} F_i(z(\omega)) - F_i(\bar{z}))) \leq \mu_2 \tag{3}$$

Other expressions may be based on the expected definite losses or a better local approximation of the front (for example a convex representation for convex multi-objective problems).

Note that for gradient-based optimization the problem has to be transformed to make the constraint functions continuously differentiable. This can be done by either reformulating the problem with the help of additional variables or by approximating the maximum function.

Figure 2 depicts the signed distance functions for the different approaches. In the two-phase approach the signed distance function is built using for example linear splines for approximating the Pareto optimal front. The expected possible losses and the linear approximation are always obtained locally for the respective deterministic outcome.

2.4 *Uncertainty Quantification*

There exist different methods to propagate uncertainties ω in the model. We make use of a non-intrusive polynomial chaos approach, which is also referred to as pseudo-spectral approach. In this approach the stochastic objective function is expanded in terms of polynomials Φ_i that are orthogonal with respect to the probability density function of the input random variables $z(\omega)$, such that

$$f(y,u,z(\omega)) = \sum_{i=1}^{\infty} \hat{f}_i(y,u)\Phi_i(z(\omega)), \tag{4}$$

with $\hat{f}_i(y,u) = \gamma_i^{-1}\text{Exp}(f(y,u,z(\omega))\Phi_i(z(\omega))$ and $\text{Exp}(\Phi_i\Phi_j) = \gamma_i\delta_{ij}$.

When applied to find statistical quantities the infinite expansion is truncated. The Fourier coefficients are approximated by first using stochastic collocation with quadrature points and then employing a quadrature rule that is suitable for the used polynomials.

3 Multi-Objective Optimization

3.1 *Constraint-Based Approach*

The formulation of robust Pareto optimal solutions results in a multi-objective optimization problem. We solve it by using the ε-constraint method.[5] The concept of this method is to optimize one objective function f_{s_j} while imposing inequality constraints on the remaining competing objective functions. For the robust multi-objective optimization the constraint function is a statistical quantity. The constraints $f_i^{(j)}$ as well as the objective function f_{s_j}, that is to be optimized, are varied in the steps of the algorithm to find different Pareto optimal solutions that are evenly distributed. The resulting minimization problem for the j-th step of the algorithm applied to a general multi-objective PDE-constrained optimization problem is

$$\begin{aligned} \min_{y,u} \quad & f_{s_j}(y,u) \\ \text{s.t.} \quad & c(y,u) = 0, \\ & f_i(y,u) \le f_i^{(j)} \quad \forall\, i \in \{1,...,k\} : \; i \neq s_j. \end{aligned} \tag{5}$$

The inequality constraints for the different steps are distributed equidistantly. The outlines of the front can be found by minimizing the objective functions individually without imposing additional constraints. It can be shown that all unique solutions to the resulting single-objective optimization problem (5) are globally Pareto optimal for any upper bound $f_i^{(j)}$.[6]

3.2 *Global Optimization Method*

The correct choice of the algorithm for solving the single-objective optimization problems (5) that result from the ε-constraint method is very important. In Kusch et al.[7] a hybrid algorithm is proposed for the single-objective optimization problems to enhance the chance of finding a global optimum and thus Pareto optimal points. The hybrid method combines the advantages of evolutionary and gradient-based methods. In a first step a genetic algorithm is applied on a Kriging surrogate model to avoid computationally expensive calculations. We make use of the software RoDeO,[8] that is adjusted to handle the given optimization constraints. The initial data acquisition is done using Latin Hypercube sampling. The Kriging model is trained in each optimization step using adaptive sampling based on the expected improvement method. Furthermore, several designs in the direction of steepest descent are included in the training set. In the second step of the hybrid algorithm a gradient-based optimization method is applied for the full model. The design found in the first step is used as a starting point for gradient-based optimization. The gradients are obtained using a discrete adjoint method based on algorithmic differentiation. The use of accurate derivative from algorithmic differentiation is especially useful for solving constrained optimization problems.

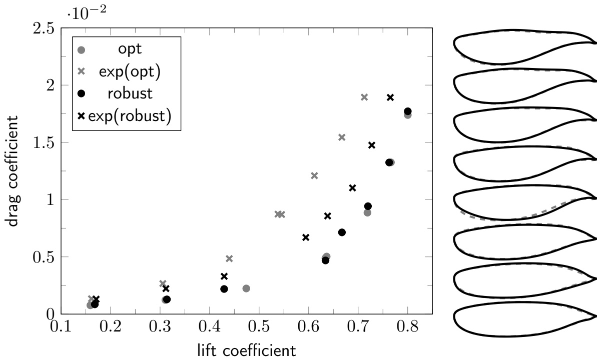

Figure 3: Pareto optimal front for two-phase approach (robust) and deterministic Pareto optimal front (opt)

4 Aerodynamic Shape Optimization

We apply the proposed method to an aerodynamic shape optimization problem for a 2D airfoil with a NACA0012 as initial design. The objective is to minimize the drag coefficient and maximize the lift coefficient. Additional inequality constraints are prescribed for the thickness of the airfoil and the resulting moment. The flow is transonic and inviscid with a Mach number of 0.8 and an angle of attack of 1.25. We assume an uncertain Mach Number, that is normally distributed $Ma \sim N(0.8, 0.01)$. The associated orthogonal polynomials for non-intrusive polynomial chaos are Hermite polynomials. The airfoil is parametrized with the help of 38 Hicks-Henne functions. The underlying steady Euler equations are solved with the open-source software SU2[9] using a Jameson-Schmidt-Turkel scheme. Gradients for the optimization in SU2 are provided by algorithmic differentiation.[10]

The two-phase approach was used with a prescribed constraint on the distance $\delta_{max} < 0.1$ in normalized objective space. Figure 3 shows the optimization result in objective function space. The dots indicate

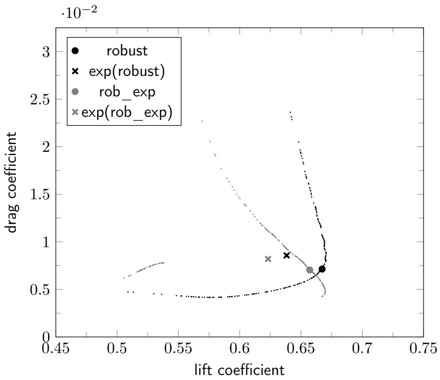

Figure 4: Sampled probability region for expected losses (robust) and the expectation-based approach (rob_exp)

the robust optimal designs evaluated for the Mach number $\bar{z} = 0.8$, that is used in the deterministic optimization. The crosses indicate the expected value. For reasons of comparison the deterministic values and the expected values of the multi-objective optimization without considering uncertainties are shown by the grey-coloured dots and crosses. The corresponding designs are plotted on the right of the figure. The upper design corresponds to the maximum lift coefficient and the lower design to the mimimum drag coefficient. It can be observed that the designs are very similar, while the expected values for the robust design approach are significantly improved.

In figure 4 random samples are shown for a chosen design to depict the probability region. The grey-coloured region is the probability region for a comparable design that was obtained using an expectation-based approach[2] with the aim to optimize the expected value of drag and lift coefficient. The probability regions differ significantly as the result obtained by the expectation-based approach leads to higher losses in objective space. In particular, the probability region based on the expected losses is close to the deterministic Pareto optimal front.

5 Summary and Outlook

We have presented a new measure for robustness when considering multiple objectives. Two approaches to include expected losses in a robust design formulation are given. A constraint-based multi-objective optimization approach making use of a hybrid method is suggested for solving the robust design problem. The approach is applied for the robust design of an airfoil. The results show that the proposed method successfully finds robust designs with less losses in objective space compared to expectation-based approaches.

For the conference we intend to apply the suggested method to a different shape optimization problem. Furthermore, we will show results for the presented direct approach for robust design with multiple objectives. In the future the aim is to include objective functions from different disciplines.

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